Propagation modeling for multimode photonics

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ABSTRACT

While ray optics is a valid model for many multimode optical systems, it cannot capture diffraction and interference phenomena which may be required, for example, in calculating speckle propagation in multimode fiber. Wave optics includes the effect of interference and diffraction, but it is usually much more costly computationally. We show how to bridge the gap between ray and wave optics by using a position-angle ray phase space representation of the electric field. By transporting a suitable quadratic functional of the field (the Wigner distribution) along rays, diffraction and interference are taken into account for propagation through an arbitrary transverse index profile. We show how this method allows us to propagate highly multimode fields without resorting to detailed mode calculations. We also illustrate the method by calculating the mode selective loss for a multimode graded index waveguide.

1. INTRODUCTION

Interest in multimode photonics systems is widespread and growing rapidly, motivated in part by ease of alignment and large numerical apertures for light collection. Examples include computer backplane communications, local area networks, and multimode interferometry. The simulation of multimode systems presents additional challenges beyond those of the single-mode case. First-principles propagation calculations can be costly except over very short distances because of the fine zoning required to accommodate a large number (thousands) of modes, and the lack of knowledge of mode-coupling perturbations. Furthermore, waveguide dispersion (absent in the single-mode case) must be assessed. Ray tracing may be able to give accurate results for energy transport but ignores phase: an accurate calculation of coupling efficiencies (e.g. VCSEL to fiber) and mode-selective loss requires capturing the details of the multimode speckle pattern. Radiation transport approaches may also be used, but this requires calculating a large number of modes and eigenvalues through WKB or some other method.

We demonstrate here the accurate propagation of speckled optical fields, without resorting to detailed mode calculations, by propagating rays in their position-angle phase space. By transporting a suitable quadratic functional of the electric field (the Wigner distribution), diffraction and interference are taken into account for propagation through an arbitrary transverse index profile. We can therefore exploit the efficiency of ray propagation over full wave optics for highly multimode systems, while retaining needed wave-optical physics. The method can be used to advantage in characterizing multimode photonics propagation and phenomena such as mode-selective loss.

This paper is organized as follows. First, we will convey the principles of the phase space method and give some simple examples. Then, we will describe the software tool we developed to study the propagation of fields through the Wigner method. This tool allows us to study the important issue of the appropriate sampling of rays in the phase space to achieve desired accuracy. Further examples of light propagation in systems with a variety of axial and transverse refractive index distributions will be given. We then show how the method can be applied to calculate modal noise in a multimode graded-index waveguiding system. Finally, we will describe some of the limitations of our current scheme and the path we envision for further development and applications.

2. BASIC FORMULATION

In this section we will develop some of the properties of the phase space formalism we will need, in an elementary way. We usually think of an electric field as a vector quantity which varies in space and time according to equations. We can also

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describe the field in terms of the wavenumbers or spatial frequencies which comprise it (Fourier representation). However, it is sometimes most natural to think in terms of a mixed representation, whereby the field is thought of as a set of spatial frequencies, the spectrum of which changes with position. For the electric field, a wavenumber defines a direction in space; the coordinates of the direction are a set of angles. Thus the field can be represented by a function of position and angle, which defines the ray phase space.

One such function is known as the Wigner distribution, introduced by E. P. Wigner in 1923[1]. The Wigner distribution allows quantum mechanical quantities to be calculated using a distribution function on a classical-like position-momentum phase space, much as we will use it to calculate wave optical properties on a position-angle phase space. Mathematically the Wigner transform can be thought of as a Fourier transform not of the field, but of its correlation function relating the field at two space points:

$$W(x,\theta) = \int_{-\infty}^{\infty} E^*(x+s/2)E(x-s/2)\exp(-ik\theta x)$$

where E is (one component of) the electric field, k is the vacuum wavenumber, and θ is the (paraxial) angle with respect to the z axis. The Wigner transform has been widely used to study not only the classical limit of quantum mechanics, but also in paraxial optics and radiation transport[2]. The usefulness of using the Wigner distribution lies in its evolution with z (for a frequency-domain paraxial problem): under multimode conditions (kL >> 1 for scale length L) or weak variation of refractive index, $W(x,\theta)$ is approximately conserved along rays:

$$W(x,\theta;z) = W\big(x_0(x,\theta;z),\theta_0(x,\theta;z)\big) + O\Big[(\lambda/L)^2\Big],$$

where the argument of W on the right hand side contains the initial conditions that lead to the phase space point (x,q) at a distance z. Furthermore, the error term is also proportional to first derivatives of the refractive index, so the propagation is exact for parabolic profiles.

An example of a Wigner distribution is shown in Fig. 1. for a Gaussian field and for a uniformly illuminated aperture. The reason diffraction is included in the ray description is that at each point in space one has a fan of ray angles: this is just a manifestation of the Fourier uncertainty principle. The Wigner distribution is not strictly a phase space density, however, since it is not always positive.

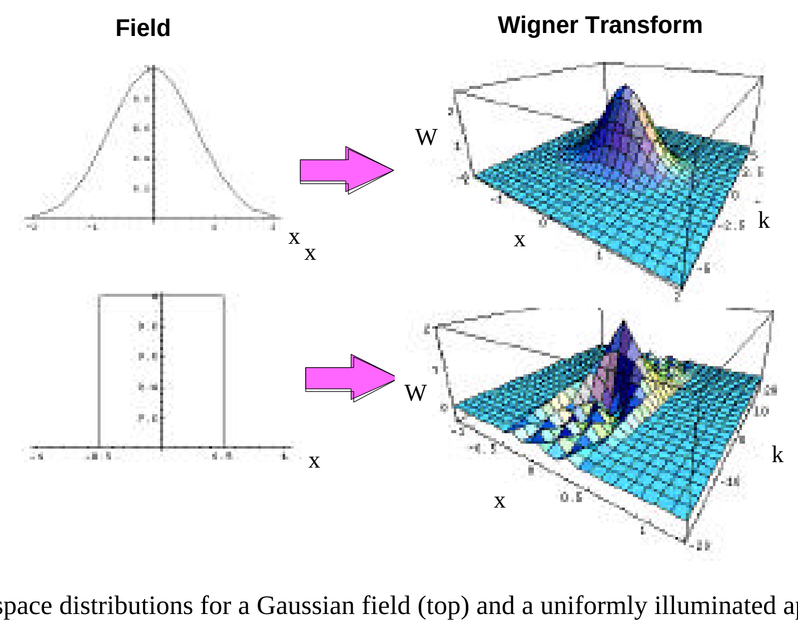

Figure 1. Wigner phase space distributions for a Gaussian field (top) and a uniformly illuminated aperture (bottom).

The Wigner propagation algorithm we have implemented works as follows. From an initial complex electric field distribution, we calculate the Wigner transform. This distribution is then evolved in space by transporting it along rays, that is, assuming conservation along a ray. As we have already indicated, this will be an excellent approximation if the index is not too rapidly varying on a wavelength scale. If at every position in the ray phase space for the new (evolved) distribution, we integrate over all ray angles, we obtain the intensity distribution as a function of spatial position ("nearfield"). Conversely, if at each angle in phase space we integrate over position, we obtain the intensity distribution as a function of ray angle ("farfield"). In practice this is usually accomplished by choosing a point or angle in the output plane and backpropagating rays to build up the integral for the intensity distributions (see Figure 2). Since in paraxial wave optics the nearfield and farfield amplitudes are related by Fourier transform, the complex electric field can be reconstructed (up to a constant phase factor) from the ray intensity distributions in position and angle.

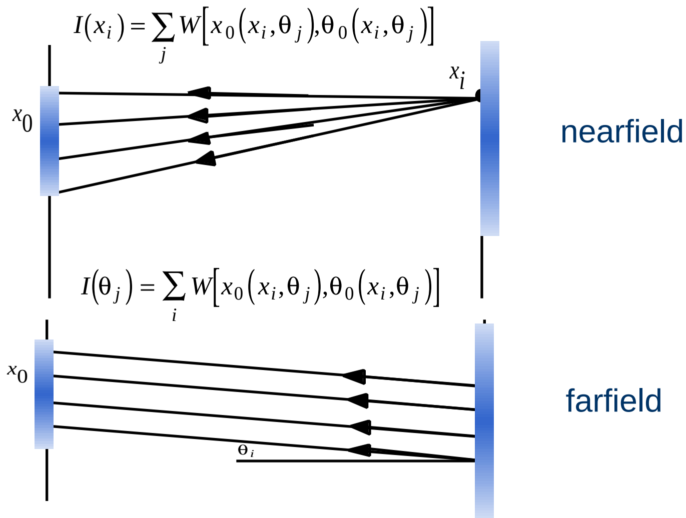

Figure 2. Illustration of the integration procedure to obtain the nearfield and farfield intensities. Rays are backpropagated from the output plane and weighted with the value of the Wigner transform at the input plane.

As an example of a calculation using the Wigner propagation algorithm, we calculated the diffraction pattern from a double slit (see Fig. 3). The Wigner function is calculated in the plane of the slits, then propagated along rays to the plane of the screen. The calculated result and the exact solution are overlaid, and agreement is excellent. In fact, for free propagation, the Wigner method is formally exact, and the only errors are due to sampling. This illustration shows strikingly that diffraction and interference, usually considered outside the domain of ray optics, can be obtained through ray tracing the Wigner distribution.

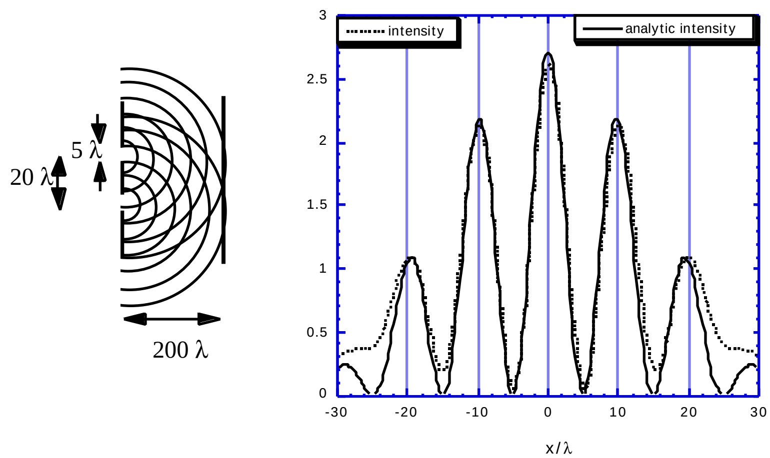

Figure 3. Wigner phase space method calculation of double-slit diffraction by ray tracing.

3. THE *PHASTER* CODE

To study the Wigner method we developed a GUI-based code which allows us to propagate beams in a variety of media in two spatial dimensions. We call the code PHASTER: Phase Space Techniques for Electromagnetics Research. PHASTER allows us to set up an arbitrary initial beam consisting of a sum of Gaussians with selected widths and amplitudes. After computing and displaying the Wigner distribution for the beam, it will solve the ray equations for a set of points in the ray phase space which sample the Wigner distribution in a prescribed manner. The rays are traced using an adaptive Runge-Kutta method through a variety of refractive index distributions having both transverse and axial variation. After propagating the prescribed set of rays to the exit plane, it will display the phase space distribution at the exit plane, as well as the x-space and angle space intensities (nearfield and farfield). PHASTER gives useful insight into the dynamics of the rays in phase space and their effect on the wave optical distribution.

Our first example of a PHASTER calculation is shown in Fig. 4. A Gaussian beam is shown propagating through three “soft slabs”, i.e., small regions where the refractive index rises and falls, depicted in the central window of the GUI. The contour plot on the left shows the initial Wigner distribution of the Gaussian. Below it we see the x-space intensity, and to the left the angle-space intensity (farfield). The windows on the far right show the analogous data for the propagated distribution. Note that the phase space distribution has become elongated and tilted: this is simply a graphic depiction of diffraction. Rays at large magnitude angles (top and bottom of distribution) move fastest in x, while small angle rays (near center of distribution) move little. The result is a spreading of the x-space distribution while the angular distribution does not change, exactly what we would expect from simple free-space diffraction. Additional spreading in x-space occurs due to the plates, but not in angle because rays receive no transverse impulse from the axially varying index.

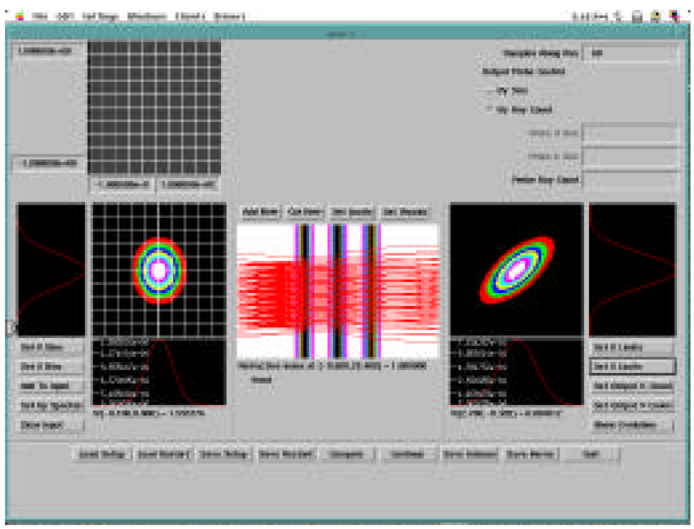

Figure 4. PHASTER calculation of propagation through three slabs. The tilted distribution on the right is the manifestation of diffraction in the ray phase space.

In Figure 5 we show the PHASTER calculation of propagation in a waveguiding structure. The rays are obviously confined to the waveguide, and at the exit have formed a galaxy-shaped distribution. A given ray will encircle the origin as it traces out a periodic path, but the frequency of the rotation decreases as the angle increases, causing the spiraling.

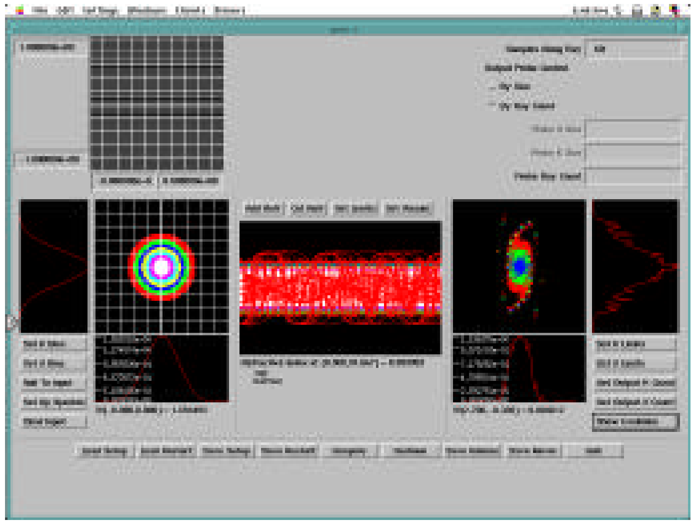

Figure 5. PHASTER propagation in a waveguiding structure. The rays are obviously confined to the waveguide, and at the exit have formed a galaxy-shaped distribution. A given ray will encircle the origin as it traces out a periodic path, but the frequency of the rotation decreases as the angle increases, causing the spiraling.

Our final example, Fig. 6, shows the propagation of a speckled beam through a linearly graded-index structure. By speckled we mean that we are launching a set of Gaussians with a distribution of transverse tilt angles and random phases. The important observation here is that we can propagate a beam with a great deal of structure, arising from the interference of many plane waves, through ray propagation in phase space.

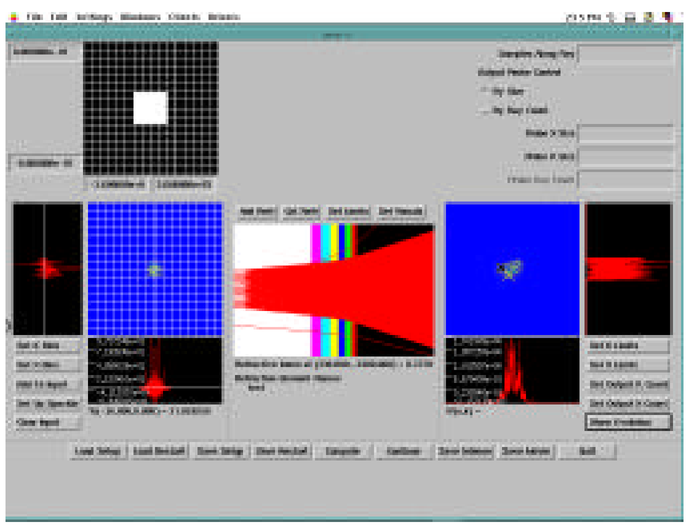

Figure 6. PHASTER calculation for a speckled beam at normal incidence on a linear index gradient

An important feature of the code is its flexibility in sampling the phase space at the entrance plane so as to put a higher density of rays where the Wigner distribution has larger values. Sampling is critical for the method, because if we do not sample wisely we suffer in computational efficiency for a given accuracy. PHASTER allows us to divide the phase space into rectangular bins and vary the size of the subdivisions of the bins.

4. APPLICATION: MULTIMODE WAVEGUIDE

In this section our goal will be to calculate the modal noise in a multimode waveguide by using the phase space dynamics of rays. Consider a graded index slab waveguide with a refractive index profile given by $n^2(x) = n_{cladding}^2 + NA^2 \operatorname{sech}^2(x/a)$, where $NA^2 = n_{core}^2 - n_{cladding}^2$. The ray trajectories in this case can be calculated analytically. We launch a Gaussian field into this waveguide; the Wigner transform of this beam is also Gaussian. In Fig. 7 we show the result of propagating this initial distribution. The phase space density twists about the origin, because the period of oscillation decreases as the initial condition moves out from the origin. Since the evolution is Hamiltonian (no gain or loss), the area (total number of rays) conserved as it stretches. In Fig 8 we show the result of projecting this distribution onto the spatial axis; this is the field intensity of a speckled beam, which shows spikes where the slope of the twisting distribution diverges. These spikes are not caustic singularities, however, as the initial distribution is smooth and so is its projection. Note that the rays for the propagated distribution are confined to the NA of the guide; rays outside have been lost.

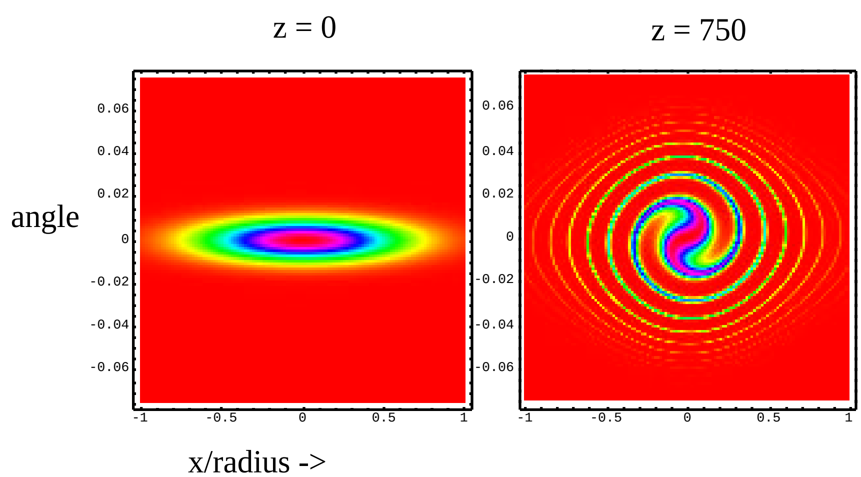

Figure 7. A Gaussian beam in phase space is propagated 750 wavelengths; the distribution spirals about the origin. The rays are confined to the waveguide NA = 0.05.

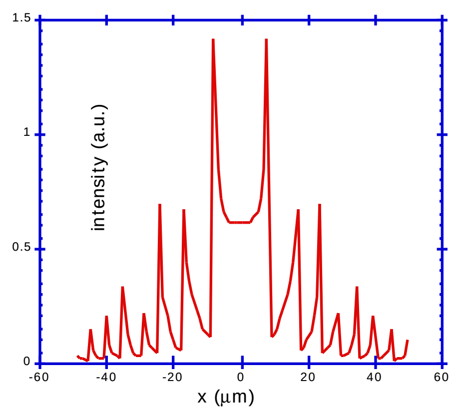

Figure 8. The phase space distribution is projected onto the spatial axis to obtain the field intensity.

As the distribution propagates in z, the position and density of the intensity spikes moves. Because the local intensity exhibits wide variation, the coupling of this waveguide to another waveguide which does not match its spatial aperture and/or numerical aperture will also be z dependent. This is a phase space manifestation of modal noise. To calculate the coupling between waveguides in the phase space formulation, we integrate the Wigner distribution of the field over the phase space of the receiving fiber, that is, over its spatial and angular aperture. If the receiving waveguide is tilted, we offset the integration limits. Thus the coupling efficiency η is given by

$$\eta(Z) = \frac{\int_{x-}^{x+} dx \int_{\theta-}^{\theta+} d\theta \; W(x,\theta;Z)}{\iint_{\Gamma} W(x,\theta;Z))},$$

where $x_{\pm} = \pm a/2 + x_{off}$, $\theta_{\pm} = \pm NA_r + \theta_{tilt}$, a is the width, x_{off} is the offset, NA_r is the NA and θ_{tilt} is the tilt of the receiving waveguide. The integral in the denominator is over all of phase space (effectively over the aperture and NA of the transmitting waveguide).

To calculate the modal noise, we can calculate the variance of η as Z is varied. Explicitly, we sample η at discrete locations along Z, starting at a distance Z_0 that is many paraxial ray oscillation periods from z = 0. Thus,

$$\sigma^2 = \frac{1}{N}\sum_{j=1}^{N} \left(\eta\left(Z_0 + j\Delta Z\right) - \bar{\eta}\right)^2,$$

where $\bar{\eta} = 1/N \sum_{j=1}^{N} \eta\left(Z_0 + j\Delta Z\right)$ and ΔZ is chosen such that $\Delta Z/\lambda >> 1$.

In Fig. 8, we show the results of a calculation of modal noise as a function of the NA of the waveguide. Here the transmitting guide width is 100λ, and the receiving waveguide width is 12.5λ. (The receiving waveguide width is chosen to be small to emphasize the noise.) We see that the noise decreases with NA, expected since the number of modes and hence speckles increases with NA. We have also plotted in Fig 8 the results of a wave optics calculation using a finite-difference Beam Propagation Method. We see that the phase space calculation agrees very well with the full wave optics code, showing that by tracing rays we can in fact accurately calculate modal noise.

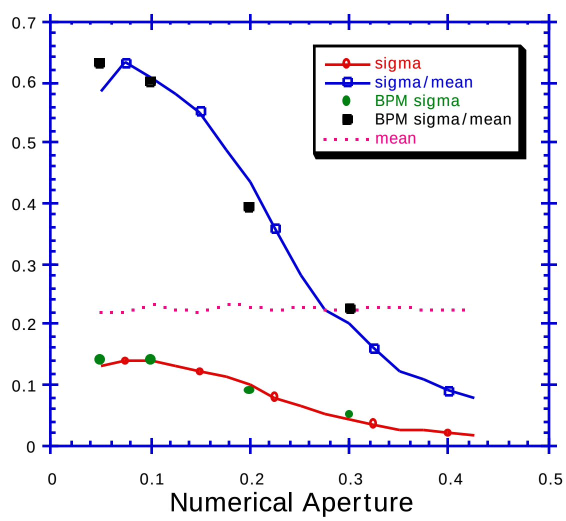

Figure 8. The variation of modal noise with numerical aperture. Standard deviation σ of coupled power is plotted for phase space method (empty markers) and using a finite-difference BPM (filled markers).

5. SUMMARY

We have presented the phase space approach as a promising method for calculating propagation in multimode photonics systems. The principal strength of the method lies in its ability to include diffraction and phase information in a ray tracing calculation: the method requires calculating the Wigner function for an optical field and simply propagating it along the geometrical ray paths. We have illustrated the method through the software tool PHASTER, and shown how phase space optics can be used to calculate accurately the modal noise in the coupling of multimode waveguides. Phase space methods will be most useful in the highly multimode case, where tracing rays is much less costly than full wave optics calculations. The method naturally has limitations: if the refractive index varies too rapidly in space, the approximation of conservation along a ray will not be well satisfied, although a higher-order method can be implemented. The method as we have presented it is limited to scalar, frequency domain paraxial optics, but these restrictions too can be relaxed. We have also limited our examples to two spatial dimensions (transverse + axial); generalization to three dimensions is straightforward, but judicious sampling in the phase space become more important. Finally, we should point out that the Wigner distribution, though widely used, represents only one choice of a phase space distribution and other functionals may also be useful[3].

We hope to have shown that a phase space picture of multimode photonics systems can be very useful in calculations, as well as gaining insight into the dynamics of the optical field.

6. ACKNOWLEDGMENTS

This work was performed under the auspices of the U. S. Department of Energy by the Lawrence Livermore National Laboratory under contract No. W-740-ENG-48.

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